On a class of quasilinear equations involving critical exponent and nonlinearity concave at the origin

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Abstract. In this talk, we are interested in solving two class of quasilinear problems, namely,

$$\left\{ \begin{array}{l} -\Delta u - u\Delta(u^2) = -\lambda|u|^{q-2}u + \mu u + (u^+)^{p-1}, \ \text{ in } \Omega, \\ u = 0, \ \text{ on } \partial\Omega, \end{array} \right\}. \tag{P}$$

and

$$\left\{ \begin{array}{l} -\Delta u - u\Delta(u^2) = -\lambda|u|^{q-2}u + |u|^{22^*-2}u + \mu g(x,u) \ \text{ in } \Omega, \\ u = 0, \ \text{ on } \partial\Omega, \end{array} \right. \tag{Q}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with regular boundary $\partial\Omega$ and $22^* = 4N/(N-2)$. For problem (P), we assume $N \geq 1$, $\lambda, \mu > 0$, $1 < q < 2$ and $4 < p < 22^*$ if $N \geq 3$ or $p > 4$ if $N = 1$ or $N = 2$ and prove the existence of two nontrivial classical solutions for problem (P) for $\lambda > 0$ fixed and for any $\mu > 0$ large enough. Firstly, we make a correct change of variable $u = f(v)$, introduced in [2], which relates problem (P) with a semilinear problem that possesses two classical solutions obtained via Mountain Pass Theorem. It is also proved that the correspondent minimax levels converge to zero as $\lambda \to 0^+$. Moreover, one of the solutions is nonnegative and the other is nonpositive.

For problem (Q), we assume $N \geq 3$, $\lambda, \mu > 0$, $1 < q < 4$ and g has a subcritical growth and possesses a condition of monotonicity. Introducing a new type of a Nehari set, more specifically, defining

$$\mathcal{N} = \left\{ w \in H_0^1(\Omega) \setminus \{0\};\ I'_{\lambda,\mu}(w)\frac{f(w)}{f'(w)} = 0 \right\},$$

where $I_{\lambda,\mu}$ is the energy functional associated with problem (Q) and f is the change of variable, we note that every critical point of $I_{\lambda,\mu}$ is contained in $\mathcal{N}$ and prove that one of the following cases is valid: 1. Problem (Q) has three solutions, one of which is nodal and ground state solution, one is nonnegative and the other is nonpositive; 2. Problem (Q) has two solutions, one of which is nonnegative and ground state solution and the other is nonpositive;
3. Problem (Q) has two solutions, one of which is nonpositive and ground state solution and the other is nonnegative, whenever $\lambda > 0$ is fixed and $\mu > 0$ is large enough. With ground state solution, we mean a nontrivial solution $u_0 \in H_0^1(\Omega)$ that satisfies $I_{\lambda,\mu}(u_0) = \inf_{\mathcal{N}} I_{\lambda,\mu}(w)$.

In order to prove the existence result for problem (Q), some difficulties naturally arise. One of them is that we had to pass for a Brezis-Lieb Lemma applied in a term that depends on f, the change of variable. So we studied the growth of the change f, what allowed to use Theorem 2 in [1] (general Brezis-Lieb Lemma) to overcome this difficulty.

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References

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